Learning Logically Defined Hypotheses

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Outline

I. A Declarative Model-Theoretic Framework for ML

II. First-Order Hypotheses on Low-Degree Structures
(joint work with Martin Ritzert)

III. Monadic Second-Order Hypotheses on Strings
(joint work with Christof Löding and Martin Ritzert)

IV. Concluding Remarks

A Declarative Model-Theoretic Framework for ML

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Declarative approach

Try to separate model from solver as far as possible.

Idea of Model-Theoretic Framework

Background structure

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Parametric model

Model described by formula of a suitable logic, which usually has certain free variables for parameters.

Example 1

Goal

Try to predict chances on academic job market based on publication data.

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Data

A list of applicants, or rather certain pieces of information about the applicants, labelled by the information of whether they succeeded or not.

Example 1 (cont'd)

Scenario 1

Suppose for each person we only have the following information:

$$p = \text{number of publications}, \qquad t = \text{years since PhD}.$$

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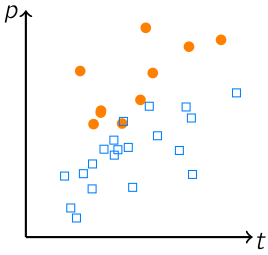

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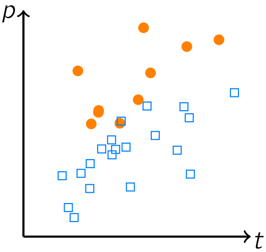

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Linear model with two parameters a, b:

$$p \geq at + b.$$

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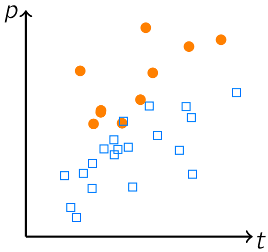

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Representation in our framework

- Background structures: ordered field of the reals
- Model: $\varphi(x_1, x_2\,; y_1, y_2) := (x_1 \geq y_1 \cdot x_2 + y_2)$

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Scenario 2

We have a publication database of a schema that includes relations

- AUTHOR(auth-id, name, affill)
- PUB(pub-id, auth-id, title, journal, year, . . .)

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Our model may say something like the following (with parameters $a, b, c_1, \ldots, c_m, d_1, \ldots, d_n$):

- the candidate has at least a publications on average per year
- and at least b single-author publications
- and either a joint publication with an author from one of the universities $c_1, \ldots, c_m$
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This can be expressed by a SQL query $\varphi(x\,; y_1, \ldots, y_{m+n+2})$.

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Select all positions with letter 'B' in LaTeX math mode.

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Hypotheses

For each parameter tuple $\bar{v} \in U(B)^\ell$ a Boolean function $[\![\varphi(\bar{x}\,;\bar{v})]\!]^B \colon U(B)^k \to \{0,1\}$ defined by

$$[\![\varphi(\bar{x}\,;\bar{v})]\!]^B(\bar{u}) := \begin{cases} 1 & \text{if } B \models \varphi(\bar{u}\,;\bar{v}), \\ 0 & \text{otherwise.} \end{cases}$$

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- Usually, only a small part of of the background structure can be inspected at runtime
- At this point it is wide open what may constitute good logics for specifying models.
- Approach probably best suited for applications where specifications in some kind of logic or formal language are common, such as verification or database systems.

Input

Learning algorithms have access to background structure B and receive as input a training sequence T of labelled examples:

$$(\bar{u}_1, \lambda_1), \ldots, (\bar{u}_t, \lambda_t) \in U(B)^k \times \{0, 1\}.$$

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Goal

Find hypothesis of the form $[\![\varphi(\bar{x}\,;\bar{v})]\!]^B$ that generalises well, that is, predicts true target values for instances $\bar{u} \in U(B)^k$ well.

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- $\rho(H)$ only depends on φ (typically function of quantifier rank).

Often we regard φ or at least its quantifier rank fixed. Then this amounts to empirical risk minimisation (ERM).

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- This implies PAC-learnability (in an information theoretic sense).
- However, it comes without any guarantees on efficiency.

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  For example, we should be able to access the neighbours of a vertex in a graph.

Complexity Considerations

- We strive for algorithms running in time polynomial in the size of the training data, regardless of the size of the background structure (or at most polylogarithmic in the size of the background structure).

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- Then we can simply ignore the regularisation term (only depending on φ) and follow the ERM paradigm:
  we need to find a formula of quantifier rank at most q and a parameter tuple that minimise the training error.

First-Order Hypotheses on Low-Degree Structures

Theorem (G., Ritzert 2017)

There is a learner for FO running in time

$$(d + t)^{O(1)}$$

where

- $t = |T|$ *is the length of the training sequence*
- d *is the maximum degree of the background structure* B
- *the constant hidden in the* $O(1)$ *depends on* q, k, ℓ.

Proof

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Exploit locality of FO (Gaifman's Theorem).

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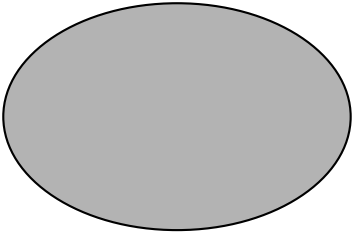

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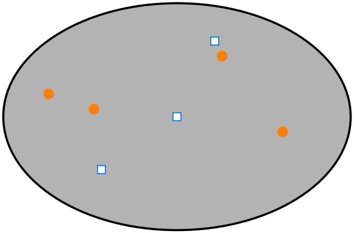

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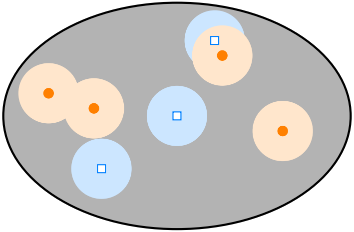

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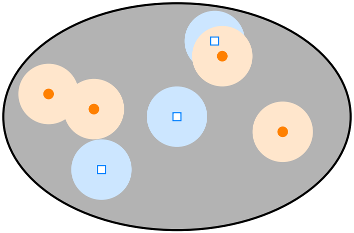

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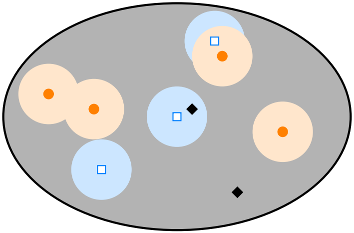

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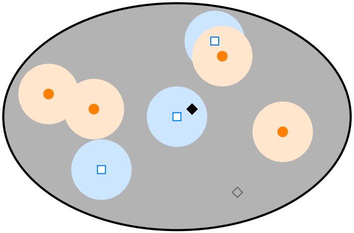

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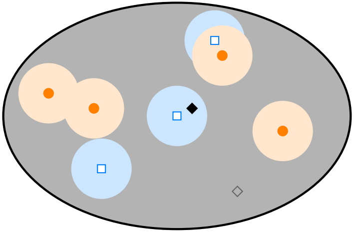

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Algorithm

Search through all local formulas of desired quantifier rank and all parameter settings close to training points and check which hypothesis has the smallest training error.

Monadic Second-Order Hypotheses on Strings

Strings as Background Structures

String $a_1 \dots a_n$ over alphabet Σ viewed as structure with

- universe $\{1, \dots, n\}$,
- binary order relation $\leq$ on positions,
- for each $a \in \Sigma$ a unary relation R_a that contains all positions i such that $a_i = a$.

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Example

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Formula

$$\varphi(x\,;y) = R_a(x) \wedge \exists z\Big(z < x \wedge \forall z'\big(z < z' < x \to R_a(z')\big) \wedge \big((R_b(z) \wedge z < y) \vee (R_c(z) \wedge z \geq y)\big)\Big)$$

with parameter $v = 35$ consistent with training examples.

Learning with Local Access

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Theorem (G., Löding, Ritzert 2017)

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Theorem (G., Löding, Ritzert 2017)

1. *There are learners running in time $t^{O(1)}$ for quantifier-free formulas and 1-dimensional existential formulas over strings.*

2. *There is no sublinear learning algorithm for $\exists\forall$-formulas or 2-dimensional existential formulas over strings.*

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Bummer

Previous theorem shows that learning MSO (even full FO) is not possible in sublinear time.

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Example

$$\ldots b aaaaaaaaaaaaaaaaaaaa\underline{a}aaaaaaaaaaaaaaaaaaa c \ldots$$

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Example

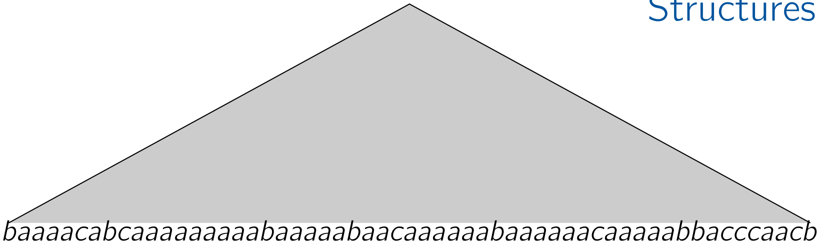
$$\ldots baaaaaaaaaaaaaaaaaaaa\underline{a}aaaaaaaaaaaaaaaaaaac \ldots$$

Solution: Index on Background Structure

We can resolve this by building an index data structure over the background string.

We do this is a pre-processing phase where we only have access to the background structure, but not yet the training examples.

Factorisation Trees as Index Data Structures



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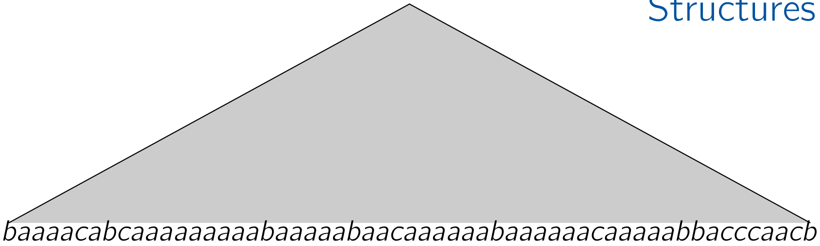

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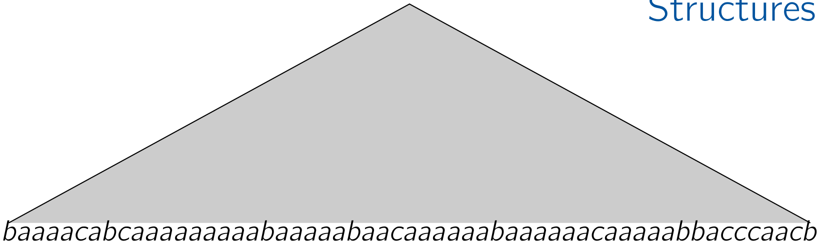

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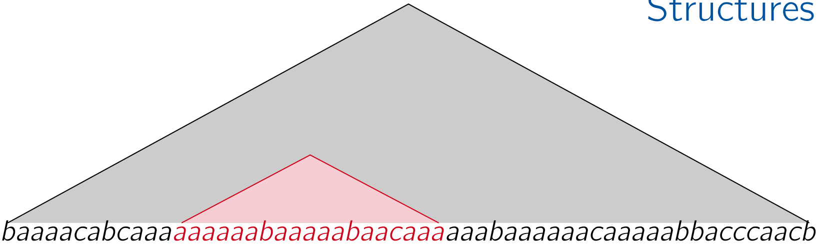

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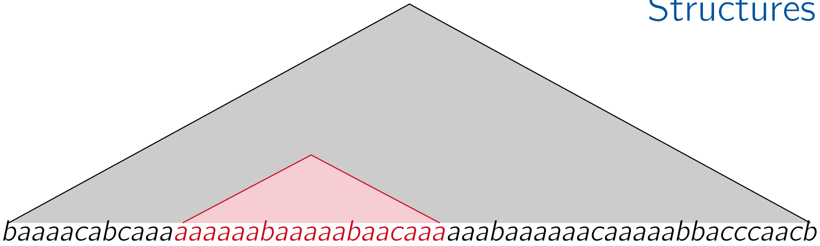

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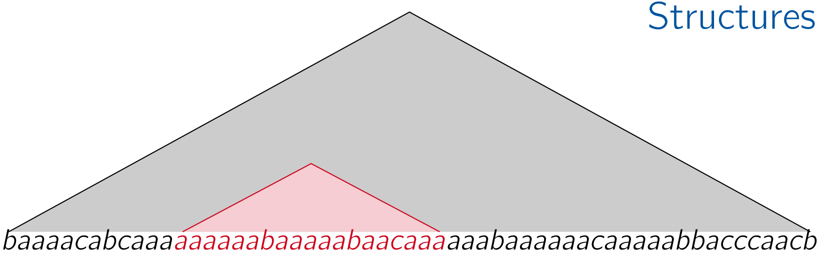

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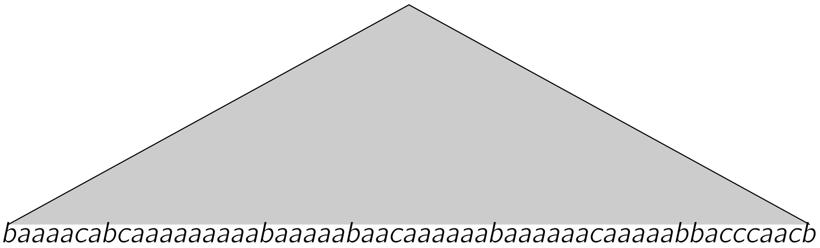
We can construct a factorisation tree of constant height for a given string in linear time (where the constant depends non-elementarily on the quantifier rank q).

Learning MSO

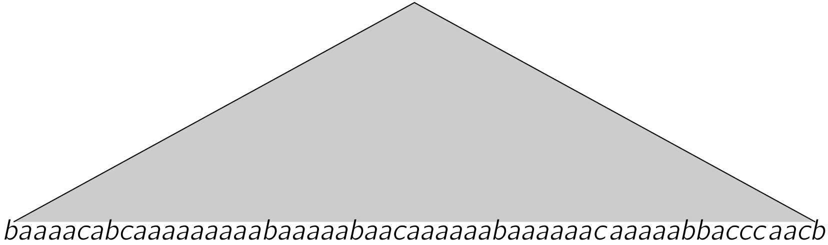
Theorem (G., Löding, Ritzert 2017)

There is a learner for MSO over strings with pre-processing time $O(n)$ *and learning time* $t^{O(1)}$.

In the pre-processing phase, our algorithm builds a Simon factorisation tree for the background string B.



One by one, the training examples are incorporated into the factorisation tree.



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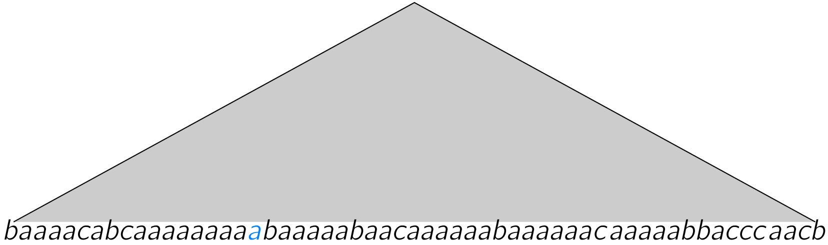

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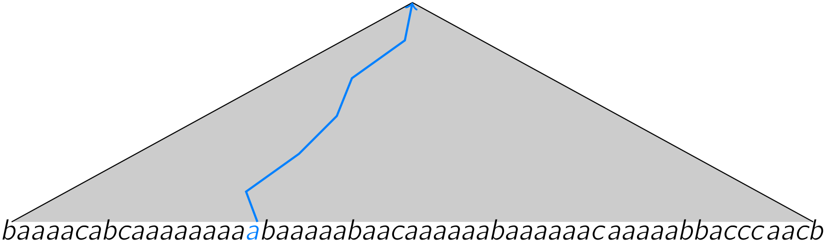

To process a new example, we need to follow a path to the root an re-structure the tree along the way. The height of the tree may increase by an additive constant.

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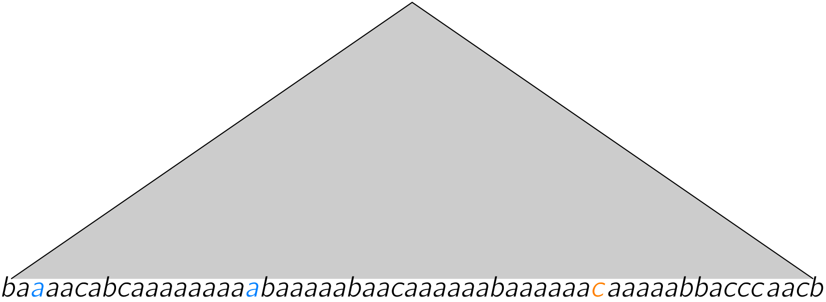

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Learning Phase 2

To find a suitable choice of parameters, one has to process the tree in a top-down manner along branches from the root to the leaves (one branch per parameter).

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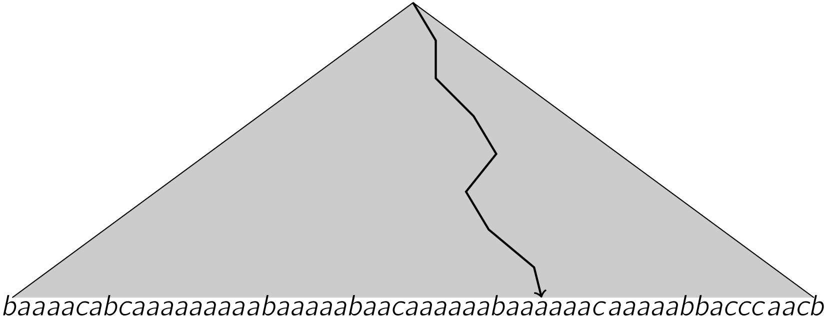

Where do we go from here?

Open Problems

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- What are suitable logics anyway?
- Go beyond Boolean classification.
- Can we design practical learning algorithms for our framework?

Vision

Design an data analysis system much like a databases system, providing an interface to “predictive queries” and for querying complex ML models (like ANNs).

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